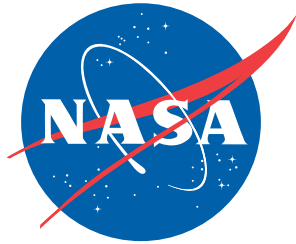


NASA/TM-2009-215749  
NESC-05-57/04-027-I



# Support Provided to the External Tank (ET) Project on the Use of Statistical Analysis for ET Certification Consultation Position Paper

*Cynthia H. Null/NESC  
Ames Research Center, Moffett Field, California*

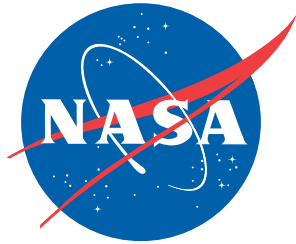
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National Aeronautics and  
Space Administration


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May 2009

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**Support Provided to the External Tank (ET) Project on the Use of  
Statistical Analysis for ET Certification  
Consultation Position Paper**

Prepared by:  
Cynthia Null  
NESC Statistics Consulting Team Lead

May 26, 2005

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## Signature Page

### NESC Technical Evaluation Team

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Dr. Cynthia H. Null


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Dr. Jeffrey D. Scargle, ARC

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Dr. Bruce Conway



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## 1.0 Executive Summary

In June 2004, the June Space Flight Leadership Council (SFLC) assigned an action to the NASA Engineering and Safety Center (NESC) and External Tank (ET) project jointly to characterize the available dataset [of defect sizes from dissections of foam], identify resultant limitations to statistical treatment of ET as-built foam as part of the overall thermal protection system (TPS) certification, and report to the Program Requirements Change Board (PRCB) and SFLC in September 2004. The NESC statistics team was formed to assist the ET statistics group in August 2004.

Based on previous flight history and early destructive analysis (dissections), decisions were made to remove manually sprayed foam from several closeout areas. These “remove and replace” areas have foam reapplied using enhanced processes and more rigorous controls.


Approximately 95 percent of the foam is being left on the tanks, as it was originally sprayed, with no rework or repair. These areas consist mainly of “acreage foam” applied with automated processes, but a large number of manually sprayed areas will also be left “as is”.

In its simplest terms, the certification approach taken by the ET project for the debris requirement (cohesive failure mode) is to determine, by testing, those combinations of defect size, shape, and depth within the foam which can cause a divot (foam debris).

The joint statistical team defined the data available, specified the assumptions that must be made to establish statistical descriptions of the dissection data, and defined limitations on using statistics for certification. Choosing a distribution to represent the observed dissection data is a critical step in the statistical analysis. There is currently no engineering rationale to choose one distribution over another. Several distributions may fit the data equally well, and provide very different estimates of the expected maximum defect size.

In addition, the NESC technical consultation team concluded that:


1. The largest characteristic defect size (CDS) value in the next sample is the quantity of interest, and should be estimated from the distribution of maximum values.
2. Using the distribution of sample values in place of the distribution of the maximum values of a sample can raise the error probability by orders of magnitude.
3. The critical information needed to predict maximum CDS for the next ET is the shape of the upper tail of the distribution of defect sizes.

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Adoption of a standard distribution cannot be justified, so this shape is unknown and cannot be effectively constrained by the existing dissection data. Based on these findings, the technical consultation team found that there is no firm basis for using statistical analysis of the dissected data from ET 94 to serve as a statistical distribution model for the certification of “as-built foam” for other tanks.

## 2.0 Identification

<b>A/I/C #:</b> 04-027-I	
<b>Initiator Name:</b> Randy Galloway	<b>Initiator Contact Info:</b> <a href="mailto:Thomas.R.Galloway@nasa.gov">Thomas.R.Galloway@nasa.gov</a> 228-332-3281
<b>Short Title:</b> Technical Consultation of the Statistical Analysis Support Provided for the External Tank (ET) Project. Note: After the NRB presentation on May 26, 2005, it was recommended to change the title to <i>Support Provided to the External Tank (ET) Project on the Use of Statistical Analysis for ET Certification</i>	
<b>Description:</b> The June 2004 Space Flight Leadership Council (SFLC) assigned an action to the NASA Engineering and Safety Center (NESC) and the External Tank (ET) project to characterize the available dataset, identify resultant limitations to statistical treatment of ET as-built foam as part of the overall Thermal Protection System (TPS) certification, and report to the PRCB and SFLC.	
<b>Date Received:</b> June 2004	<b>Date A/I/C Initiated:</b> June 2004
<b>Initial Evaluators Assigned:</b> NA	<b>Initial Evaluators Contact Info:</b> NA
<b>Lead Assigned:</b> Cynthia H. Null	<b>Lead Contact Info:</b> Cynthia.H.Null@nasa.gov 650-604-1260

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## 2.1 Team Members, Consultants, and Advisors

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### Advisors:


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## 3.0 Consultation Plan

The following are the major activities of the NESC consultation:

Milestone	Date
Consultation Initiation	June 2004
Team Formation	August 2004
Technical Interchange Meeting	4-6 August, 2004
Evaluation	August – September 2004
Stakeholder Out Briefing	26 September 2004
Final Report Submission	16 May 2005

The NESC consultation team met with the ET statistics group August 4-6, 2004 at the Michoud ET Assembly plant, for briefing and discussion of the dissection data. The NESC team was provided with the dissection data as well as already completed analyses of the dissection data. Through telecons and face-to-face meetings, the joint statistics team defined the data available, specified the assumptions that must be made to establish statistic descriptions of the dissection data, and defined limitations on using statistics for certification. Dr. Fayssal Safie presented the joint finding to the PRBC on September 30, 2004 (see Appendix B).

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In addition, the NESC team did independent analyses of the dissection data to establish their conclusions.

## 4.0 Description of the Problem and Proposed Solutions

### 4.1 Problem


Choosing a distribution to represent the observed data is a critical step in the statistical analysis. There currently is no engineering rationale to choose one distribution over another. Several distributions may fit the data equally well, and provide very different estimates of the expected maximum. The ET program has identified four foam failure modes. Understanding the frequency and size of internal defects has become important to understanding and predicting debris release.

The ET has several different types of closed cells foam, various ablator materials, and ice /frost that form potential contributors to the debris environment around the Space Shuttle on ascent. For the return to flight (RTF) certification effort, the major focus has been on the areas of manually sprayed foam which were shown to be highly susceptible to having latent defects (voids) within the foam that could cause structural failure of the surrounding foam due either to entrapped gas pressure or liquid nitrogen /liquid air gasification within the voids on ascent.

Based on previous flight history and early destructive analysis (dissection), decisions were made early in the RTF process to remove manually sprayed foam in the area of the bipod fitting and the intertank-to-liquid-hydrogen-tank flange. Since that time, other areas were added to the “remove and replace” category based on data acquired in the RTF process. These include the forward 10 feet of the Protuberance Airload (PAL) ramp on the liquid hydrogen tank and the longeron areas at the aft end of the tank. These “remove and replace” areas are having foam reapplied using enhanced processes and more rigorous process controls. The manually applied foams include BX-250 and BX-265. BX-265 replaced BX-250 due to environmental issues with the blowing agent in BX-250. Generally speaking, BX-250 is installed on tanks through ET-120, and BX-265 is installed on existing tanks subsequently processed (ET-121 and forward). BX-265 is being applied in all redesign/rework areas.

Approximately 95 percent of the foam is being left on the tanks as it was originally sprayed with no rework or repair. These areas consist mainly of “acreage foam” applied with automated processes, but a large number of manually sprayed areas will also be left “as is”.

In its simplest terms, the certification approach taken by the ET project for the debris requirement (cohesive failure mode) is to determine, by test, those combinations of void (defect) size /shape and depth within the foam which can cause a divot. This test program is referred to

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as “critical defect testing.” Defects of “slot” and “cylindrical” shape are tested, based on principles of fracture mechanics.

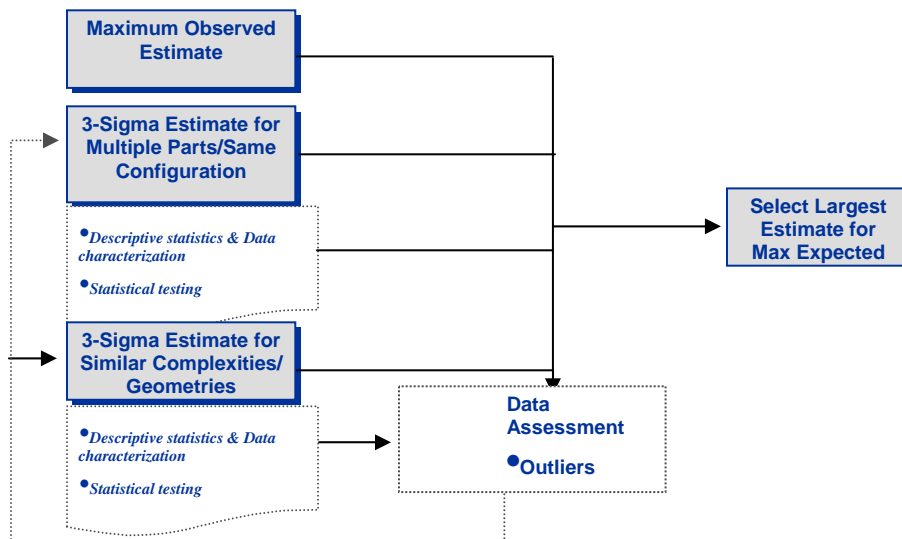
In order to determine whether an area of foam (referred to as a “closeout”) complies with the debris mass requirement, an analysis must be undertaken to determine the *process yield* of defects from that area. If the process yield defect size is less than the critical defect size (i.e., regardless of depth within the foam, no critical mass debris will be generated), then the area would be considered acceptable. If the process yield exceeds the critical defect size (i.e., at a certain range of depths, it would be possible to liberate debris exceeding the allowable limit) then further assessments that include local geometry, foam thicknesses, and thermal profiles during ascent (e.g., an area that stays cold throughout ascent can have a much larger critical defect size without producing a divot than other areas) may be required to determine if the situation is acceptable.

A survey of the technical community would undoubtedly conclude that the most intractable issues for certification of the ET for debris lie in the area of “as-built” foams. Knowledge must be inferred, either by statistical or engineering methods, from destructive evaluation of one tank (ET-94), with a few areas from other tanks.

## **4.2 Proposed Statistical Methods for ET Certification by ET Program Summer 2004.**

The proposed statistical method during Summer 2004 is outlined in Figure 4.2-1 (provided by ET program). The 3-sigma estimates of ET foam defect size are all point estimates—distribution to be determined. All defects were divided into two types, slots and cylinders, depending on the ratio of the largest (L1) and second largest (L2) dimension of the defect. If  $L1/L2 > 2.5$  a defect is called a slot, all others are called cylinders. In addition, the defects were labeled by process type (elongated cell, rollover, delamination, gunspit, void). This method assumed that datasets are primarily separated by the part that is sprayed (PAL ramp, Liquid Oxygen (LOX) intertank flange, bipod, longeron, etc.).

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**Figure 4.2-1. Three-Estimate Approach – Summer 2004**


### 4.3 New Statistics Approach Proposed in October 2004

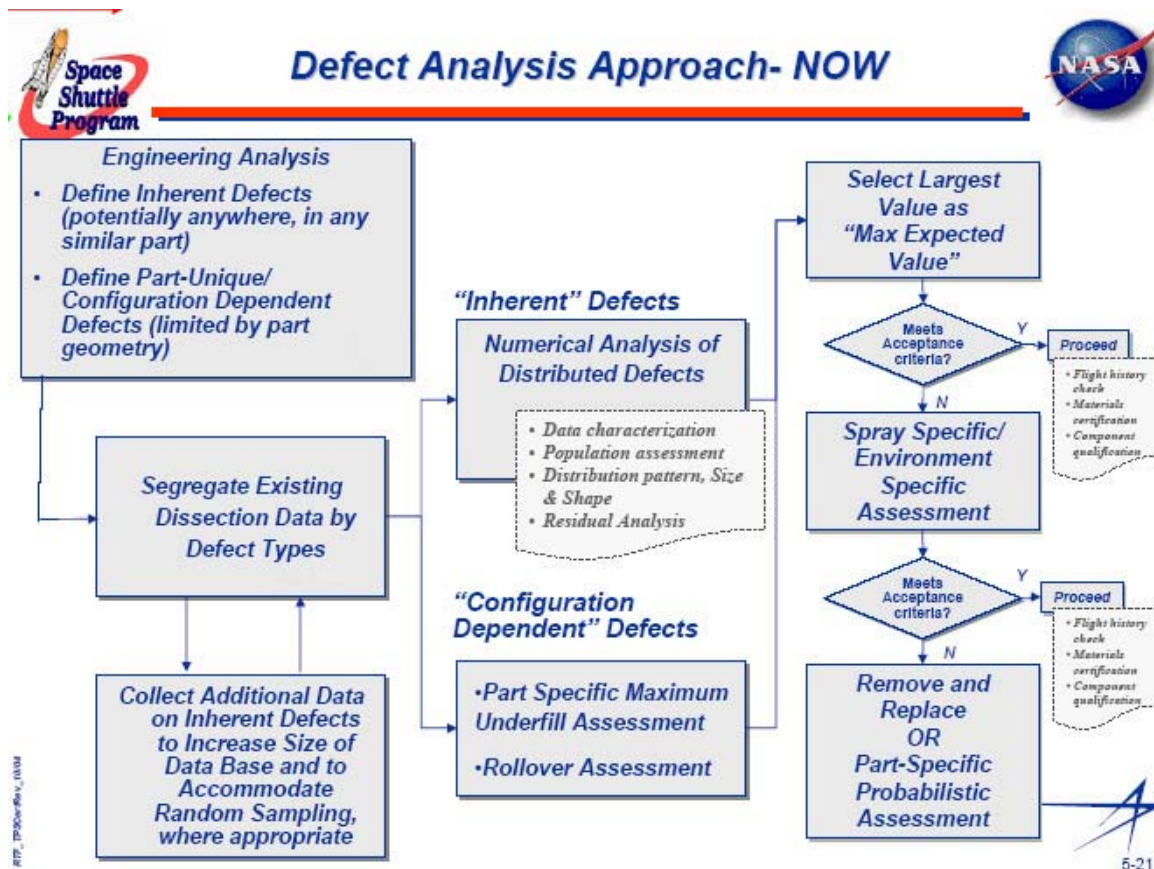
For this analysis two types of defects are defined:

1. Defects inherent to the process = “Inherent”. (If a defect can be assumed to be in potentially any part, numerical treatments will be used to assess possible size and likelihood.)
2. Defects unique to a particular configuration = “Configuration Dependent”. (Specific defect risks are determined by part geometry.

Each part will be assessed for areas prone to defect generation.) Figure 4.3-1 outlines the new approach (provided by the ET program).




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**Figure 4.3-1. New Statistics Approach – October 2004**

Distributions were fitted to the data and 3-Sigma values were derived for the best-fit distribution and for the distribution with the biggest 3-Sigma (of distributions considered). These values are point estimates that control only the probability that a single defect chosen at random will not exceed that size. Frequency of the defects in flight is not considered.

For each defect estimate for a part, the determination of an expected maximum begins with the 3-Sigma from the grouped data. This estimated can be increased by information from part-specific geometry or reduced to accommodate any hardware/ foam thickness constraints.

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## 5.0 Evaluation of Use of Statistics for Certification Approach

### 5.1 Joint Evaluation by ET Program and NESC

#### What dissection data is available?

The ET program goal is to make inferences about defects on the next tank to be flown—to support the certification of as-built foam—from the dissection data that has been collected from tank 94, and selected areas from tanks 120, 121, and 123.

Each defect is measured in three dimensions – length, width and thickness. These are sorted by largest (L1), 2<sup>nd</sup> largest (L2) and smallest (L3). Based on fracture mechanics principles, all defects were divided into two types, slots and cylinders, depending on the ratio of the largest (L1) and second largest (L2) dimension of the defect. If  $L1/L2 > 2.5$  a defect is called a slot, all others are called cylinders. If the defect is categorized as a slot, the CDS of the slot is defined to be the smaller dimension, L2; if the defect is a cylinder, the CDS is equal to the largest dimension, L1.


There are several types of void defects – elongated cells, voids, rollovers, delaminations, gunspits, etc. Void defects are further classified by the type of foam used – BX or PDL. Elongated cells typically are small voids that were judged to be primarily a function of foam chemistry, whereas the other defects are primarily a function of the spray process. In order to concentrate the analysis on defects due to the spray process, most elongated cells were purged from the database by removing defects with  $L1 < 0.5''$  and  $L2 < 0.3''$  (i.e., defects with any one dimension  $\geq 0.5''$  or any two dimensions  $\geq 0.3''$  were kept). Also, void defects from the repair of production acceptance test areas were not included.

The available data were “conveniently”, not randomly sampled, based upon tanks that were available for dissection. Since the data were not randomly selected and are available from only a few tanks, derived statistics are valid only for the sampled population and not the fleet, unless the dissected tanks are found to be typical of the fleet.

#### Available dissection data (as of September 2004)

PALRamps: 94LH<sub>2</sub>, 94LO<sub>2</sub>, 123LO<sub>2</sub>, 120LH<sub>2</sub>(10'), 121LH<sub>2</sub>(10'), 94SRB  
I/T Flanges: 94LO<sub>2</sub>, 94LH<sub>2</sub>, 120LH<sub>2</sub>, 121LH<sub>2</sub>  
Flange Thrust Panel 94LH<sub>2</sub>, 120LH<sub>2</sub>  
LO<sub>2</sub> feedline fairing: 94LO<sub>2</sub>, 120LH<sub>2</sub>, 121LH<sub>2</sub>  
Feedline Yoke: 94 (incomplete)  
Ice Frost Ramps: 94LO<sub>2</sub>, 128LO<sub>2</sub>, 94I/T, 94LH<sub>2</sub>, 120LH<sub>2</sub>



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Dog Leg C/T cover: 121LH<sub>2</sub>  
Longeron: 94LH<sub>2</sub>  
Nosecone: 94

### **What are the limitations?**

**The major assumptions that must be made to use statistical analysis of dissection data for certification include:**


- The defects found in ET-94 are representative of all defects that are produced during the manually-sprayed processes for all in-stock tanks.
- The distributions of defects, for both slots and cylinders, are homogeneous (stable) and the samples are statistically independent.

**The combined team recognized that there are major limitations to generalizing from ET-94 to other tanks with “use-as-is” foam. These include:**

- Data for some parts are available from only one tank.
- Data in some cases are a mixture of BX-250 and BX-265 material.
- Data on some parts have very small sample size (e.g., N=3).
- Samples are not random—tank 94 plus certain parts of other tanks removed in support of redesign.
- Sprayer-to-sprayer differences are not well understood.

**The combined team recognized several major limitations to fitting statistical distributions to the dissection data, including:**

- There is no *a priori* knowledge of the shape of distribution of CDS.
- There is no engineering rationale to pick a specific distribution.
- The natural variation of the process is not well understood.
- There is a lack of random samples (historical) of sufficient size to empirically select a distribution.
- Process controls related to manually-sprayed foam were related to environmental parameters and overlap time. Distributions of these process variables have been studied. However, the relationship between process control variables and defects is not known.
- The critical part for predicting the maximum CDS distribution is the upper tail, where we have the least information.

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## 5.2 Additional Evaluation by NESC Technical Consultation Team

### What is the critical statistic?

There are two random variables that can be treated statistically:

- 1) The measurement of a single foam defect, selected at random.
- 2) The measurement of N foam defects, and then selecting the maximum of the N values.


These variables are not the same, and in particular the equivalent 3-Sigma point in the distribution of (1) is not the same as that of (2). The latter is what is relevant to the basic questions posed here. The distribution of the maximum value of a set of random variables can be derived from their individual distributions. The single foam defect statistic is an underestimate of the maximum expected defect. The single foam defect statistic misleads by underestimating the probability of a large defect. See Appendix C for a description.

### What assumptions must be made to perform statistical analysis on the foam dissections from ET-94?

A key assumption that must be made is that the defects found in ET-94 are representative of all defects that are produced during the manually-sprayed processes for all in-stock tanks.

Process control was applied to very few variables, and internal defects were not considered a process failure when the as-built tanks were produced. Among the variables controlled (and documented) are room temperature and humidity, substrate temperature at the beginning of a part spray, and component temperatures and proportions. The rise-time between layers and overlap time between layers of foam were well-understood by sprayers and quality personnel, though a method for timing was not provided and records were not made. Measurements made on witness panels and from plug pulls include tensile strength, density, and pull strength. The relationship between these control variables and measurements and defect production has not been determined. In addition, sprayer to sprayer differences and how they influence defect production are not well understood.

The manually sprayed foam on test tank 94 and as-built tank 120 is predominately BX-250. The manually sprayed foam on the rest of the as-built tanks is BX-265. Whether the defects that appear in BX-250 and BX-265 are similar in frequency and size is not known. The chemical formulas for BX-250 and BX-265 vary in not only propellant, but the BX-265 formula was modified in hopes that the handling of the foam, in terms of rise-time and overlap time would remain about the same.

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Although this is described as a small change, the data about the effect of this change is only understood in terms of tensile strength, compression, and density. Whether the chemical change affects any defect production variables (number, size, defect type, defect category, location, distribution, maximum characteristic defect) has not been investigated. Since BX-250 is not available for controlled testing, this is unanswerable.

BX-265 is sprayed at a much higher temperature (BX-250 was sprayed at about 110 °F, while BX-265 is sprayed at about 155 °F). Whether the higher temperature affects defect frequency or maximum defect size is unknown. The sprayers were forced to change their arm/hand position(s) to avoid hot component feedlines resting on their hands. Whether this awkward body position changed defect production is not known. Assuming that defect frequency and size from a BX-250 tank is representative of a BX-265 tank is unjustified.

### **What statistical distribution represents the defect dissection data?**

Traditional statistical methods address the problem of characterizing the distribution of a variable from one or more samples. In some cases one has considerable *a priori* knowledge of the distribution. For example, a standard formula with a few parameters may be known to be valid. And, often the goal is to predict features of the main core of the distribution—such as its median.


The problem here is to estimate properties of the distribution of the maximum CDS, but there is no useful *a priori* knowledge about the distribution family and the key properties are features of the upper tail of the distribution—so little knowledge is gained per sample about the features.

Process control variables for manually-sprayed foam were related to environmental parameters, component temperatures, and overlap time. Distributions of these process variables have been studied. However, knowledge of the process control does not provide the requisite information, since the relationship between process control variables and defects is unknown.

Additionally, witness panels, and plug pulls from sprayed foam were analyzed—density, compressions, adhesion, and so forth. Again, the relationship between these foam characteristics and defects is unknown.

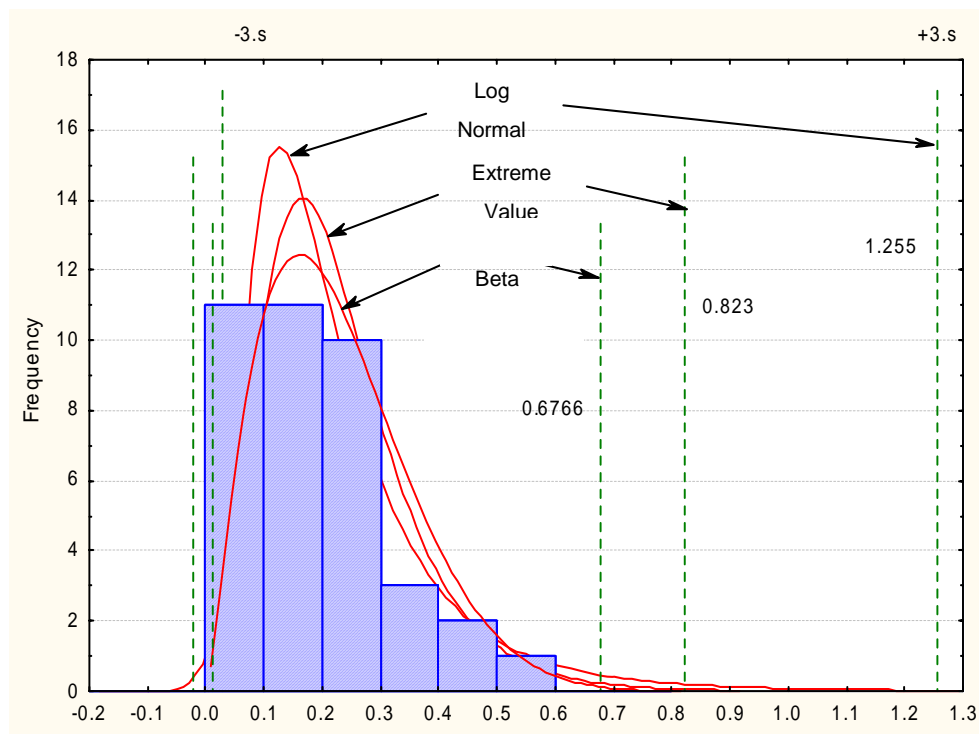
For two reasons, fitting of any of the dozens of standard distributions in the statistics literature to the dissection data provides no useful information (Note: with small samples, data in the tails of the distribution are not expected.):

1. A superior fit of one of these distributions to the core of the data provides little or no information on whether that distribution provides a more accurate description of the upper tail—the critical aspect. Standard goodness-of-fit tests are not sensitive to shape of the tail of the distribution.

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2. Various distributions with quite different tails can provide equally acceptable fits to the available datasets.

Figure 5.2-1 has three fits to one set of data of slots. Each fit is statistically acceptable (Note: the estimated 3-Sigma values are as low as .6766 and as high as 1.255). The first might be considered too small to cause a significant divot, and the latter could be above engineering limits. At this point there is no reason to pick one distribution over another.




**Figure 5.2-1. Fitting Distributions to a Sample of Dissection Data**

### **Is the 3-step largest defect approach conservative?**

In this 3-step approach, the max expected void is defined as the worst of three estimates: 3-Sigma estimate for multiple parts/same configuration, 3-Sigma estimate for similar complexities or geometries, and maximum observed estimate (See Figure 4.2-1).

This process does not assure that the estimate will always be wrong in the too large direction—despite the claims. Each of the 3-Sigma estimates is contingent on the selection of a distribution type by a goodness-of-fit test. The selection of a distribution can be erroneous (a low tail

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distribution may fit best when a high tail distribution is appropriate). With a choice of distribution varying from part to part, a statistical description of this three-step process, such as its distribution, is as yet undetermined.

### **Is the new largest defect approach an improvement?**

The same problems experienced with the old approach continue with the new approach (See Figure 4.3-1). In addition, partitioning the data into meaningful sets for analysis after the data has been collected and examined raises the concern of statistical bias. Keeping the data separate by part can be easily justified, but yields very small samples of defects. The present new partitioning of the defects into chemistry, process, and geometry or part-specific categories may be useful. However, setting the criterion for this division in an unbiased manner is difficult after the data has been examined (and does not change the fact that tank-to-tank differences are not well understood).

Although there are some defects that are unique to a part (such as a gap behind a bolt), the argument, in many cases, that a particular defect can only happen in an exact location or at a specific depth must be viewed with skepticism unless accompanied by solid data and analysis.


## **6.0 Findings and Recommendations**

### **6.1 The Combined Team Finding**

Dissection data statistics have severe limitations and significant uncertainty in predicting the size of the largest defect CDS on the next tank.

### **6.2 Recommendations to the PRCB by the Combined Team**

1. Any statistical results should be subject to an engineering evaluation for consistency with experience, engineering analysis, and hardware limitations.
2. Statistical analysis results/estimates should be used in a support role and as information to engineering in ET certification.
3. Data limitations need to be addressed and minimized to improve the credibility of the statistical analysis to support RTF decisions (data must address tank-to-tank or part-to-part variation).
4. The ET Program needs to pursue methods beside statistics to predict the largest possible defect CDS.


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### **6.3 NESC Technical Consultation Team Findings (Three-Estimate Approach)**

1. The largest CDS value in the next sample is the quantity of interest, and should be estimated from the distribution of maximum values. Using the distribution of sample values in place of the distribution of the maximum of a sample can raise the error probability by orders of magnitude.
2. The assumption that Tank 94 will have the same type, number, and distribution of defects sizes as any as-built tanks is unfounded. The assumption that BX-250 and BX-265 foam will have the same type, number and distribution of defect size is unfounded.
3. To predict maximum CDS for the next tank, the critical information needed is the shape of the upper tail of the distribution of defect sizes. Adoption of a standard distribution cannot be justified, so this shape is unknown and cannot be effectively constrained by the existing dissection data.
4. The Three-Estimate Largest Defect Approach has not been statistically justified nor assured to be conservative.

### **6.4 NESC Technical Consultation Team Findings (New Statistics Approach)**

1. The largest defect in the next sample is the quantity of interest, and should be estimated from the distribution of maximum values. Using the distribution of sample values in place of the distribution of the maximum of a sample can raise the error probability by orders of magnitude.
2. The assumption that Tank 94 will have the same type, number, and distribution of defects sizes as any as-built tanks is unfounded. The assumption that BX-250 and BX-265 foam will have the same type, number and distribution of defect size is unfounded.
3. To predict maximum defect size for the next tank, the critical information needed is the shape of the upper tail of the distribution of defect sizes. Although the amount of data has increased by combining all defects in a category, different distributions will still provide very different 3-sigma estimates and choosing a distribution is still tenuous.
4. The probabilities associated with statistical analyses are only considered defensible if the analysis methods have been described prior to the collection of the data. If the analysis has been done after the data has been examined, the analysis is considered to be suggestive. New data would be required to draw valid conclusions.

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## 6.5 NESC Technical Consultation Team Conclusion (Three-Estimate Approach)

Based on the findings above, there is no firm basis for using statistical analysis of the dissected data from ET-94 to serve as a statistical distribution model for the certification of as-built foam for other tanks.

## 6.6 NESC Technical Consultation Team Conclusion (New Statistics Approach)

The “new” approach suffers from the same issues as the “old” approach—wrong statistic (based on the distribution of defects instead of the distribution of maximums), unfounded assumptions of tank-to-tank and foam-to-foam similarity, and unfounded choice of distribution to represent the data. Therefore, there is no basis for using statistical analysis of the dissected data from ET-94 to serve as a statistical distribution model for the certification of as-built foam for other tanks.

## 7.0 Lessons Learned

N/A


## 8.0 References

N/A

## 9.0 List of Acronyms

°F	Degrees Fahrenheit
CDF	Cumulative Distribution Functions
CDS	Characteristic Defect Size
ET	External Tank
LOX	Liquid Oxygen
NESC	NASA Engineering and Safety Center
PAL	Protuberance Airload
PDF	Probability Distribution Function
PRCB	Program Requirements Change Board
RTF	Return to Flight
SFLC	Space Flight Leadership Council
TPS	Thermal Protection System




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## 10.0 Definition of Terms


Corrective Actions	Changes to design processes, work instructions, workmanship practices, training, inspections, tests, procedures, specifications, drawings, tools, equipment, facilities, resources, or material that result in preventing, minimizing, or limiting the potential for recurrence of a problem.
Finding	A conclusion based on facts established during the assessment/inspection by the investigating authority.
Lessons Learned	Knowledge or understanding gained by experience. The experience may be positive, as in a successful test or mission, or negative, as in a mishap or failure. A lesson must be significant in that it has real or assumed impact on operations; valid in that it is factually and technically correct; and applicable in that it identifies a specific design, process, or decision that reduces or limits the potential for failures and mishaps, or reinforces a positive result.
Observation	A factor, event, or circumstance identified during the assessment/inspection that did not contribute to the problem, but if left uncorrected has the potential to cause a mishap, injury, or increase the severity should a mishap occur.
Problem	The subject of the independent technical assessment/inspection.
Recommendation	An action identified by the assessment/inspection team to correct a root cause or deficiency identified during the investigation. The recommendations may be used by the responsible C/P/P/O in the preparation of a corrective action plan.
Root Cause	Along a chain of events leading to a mishap or close call, the first causal action or failure to act that could have been controlled systemically either by policy/practice/procedure or individual adherence to policy/practice/procedure.




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## **VOLUME II: APPENDICES**


### **Appendix A. NESC Request Form PR-003-FM-01**

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<b>NASA Engineering and Safety Center Request Form</b>		
Submit this ITA/I Request, with associated artifacts attached, to: <a href="mailto:nrbexecsec@nasa.gov">nrbexecsec@nasa.gov</a> , or to NRB Executive Secretary, M/S 105, NASA Langley Research Center, Hampton, VA 23681		
<b>Section 1: NESC Review Board (NRB) Executive Secretary Record of Receipt</b>		
Received (mm/dd/yyyy h:mm am/pm) 3/17/2004 12:00 AM	Status: New	Reference #: 04-027-I
Initiator Name: Randy Galloway	E-mail: thomas.r.galloway@nasa.gov	Center: SSC
Phone: (228)-688-3281, Ext _____	Mail Stop: HA00/NESC	
Short Title: Statistical analysis support for ET project		
Description: Provide support for ET project descriptive statistical regression analysis being performed to characterize current state and predict future state		
Source (e.g. email, phone call, posted on web): e-mail/phone call		
Type of Request: Support		
Proposed Need Date: 4/16/04		
Date forwarded to Systems Engineering Office (SEO): (mm/dd/yyyy h:mm am/pm):		
<b>Section 2: Systems Engineering Office Screening</b>		
<b>Section 2.1 Potential ITA/I Identification</b>		
Received by SEO: (mm/dd/yyyy h:mm am/pm): 3/18/2004 12:00 AM		
Potential ITA/I candidate? <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No		
Assigned Initial Evaluator (IE):		
Date assigned (mm/dd/yyyy):		
Due date for ITA/I Screening (mm/dd/yyyy):		
<b>Section 2.2 Non-ITA/I Action</b>		
Requires additional NESC action (non-ITA/I)? <input checked="" type="checkbox"/> Yes <input type="checkbox"/> No		
If yes:		
Description of action: Provide statistical expertise to ET project		
Actionee: Dawn Schaible		
Is follow-up required? <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No If yes: Due Date:		
Follow-up status/date:		
If no:		
NESC Director Concurrence (signature):		
Request closure date:		
<b>Section 3: Initial Evaluation</b>		
Received by IE: (mm/dd/yyyy h:mm am/pm):		
Screening complete date:		
Valid ITA/I candidate? <input type="checkbox"/> Yes <input type="checkbox"/> No		
Initial Evaluation Report #: NESC-PN-		
Target NRB Review Date:		

	<b>NASA Engineering and Safety Center (NESC) Technical Consultation Position Paper</b>	Document #:	Version:
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<b>Section 4: NRB Review and Disposition of NCE Response Report</b>			
ITA/I Approved: <input type="checkbox"/> Yes <input type="checkbox"/> No		Date Approved: _____	
		Priority: - Select -	
ITA/I Lead: _____, Phone (____) _____ - _____, x _____			
<b>Section 5: ITA/I Lead Planning, Conduct, and Reporting</b>			
Plan Development Start Date: _____			
ITA/I Plan # NESC-PL-_____			
Plan Approval Date: _____			
ITA/I Start Date	Planned: _____	Actual: _____	
ITA/I Completed Date: _____			
ITA/I Final Report #: NESC-PN-_____			
ITA/I Briefing Package #: NESC-PN-_____			
Follow-up Required? <input type="checkbox"/> Yes <input type="checkbox"/> No			
<b>Section 6: Follow-up</b>			
Date Findings Briefed to Customer: _____			
Follow-up Accepted: <input type="checkbox"/> Yes <input type="checkbox"/> No			
Follow-up Completed Date: _____			
Follow-up Report #: NESC-RP-_____			
<b>Section 7: Disposition and Notification</b>			
Notification type: - Select -		Details: _____	
Date of Notification: _____			
Final Disposition: - Select -			
Rationale for Disposition: _____			
Close Out Review Date: _____			

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### Form Approval and Document Revision History

Approved: _____ <div style="text-align: center;">NESC Director</div>	_____ <div style="text-align: center;">Date</div>
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Version	Description of Revision	Office of Primary Responsibility	Effective Date
1.0	Initial Release	Principal Engineers Office	29 Jan 04

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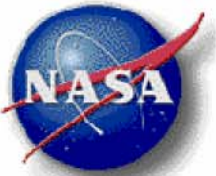
## Appendix B. Joint Presentation to PRCB



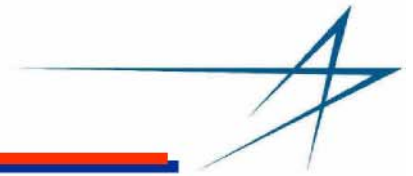
# Statistical Treatment and Limitations for the ET As-built TPS Foam

September 24, 2004





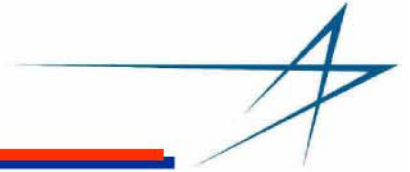
## Objective



- ***June Space Flight Leadership Council assigned an action to NESC and ET project jointly to:***
  - 1) Characterize available dataset**
  - 2) Identify resultant limitations to statistical treatment of External Tank as-built foam as part of overall TPS certification and**
  - 3) Report to PRCB and SFLC.**



# ***Statistical Treatment/Analysis***



- ***Goal***

- Assess the risk that somewhere on the "next" tank there will be a defect large enough to divot

- ***Statistical treatment***

- Defect data Characterization
  - Descriptive statistics
  - Fitting distribution to data
- Extrapolation
  - Estimation of the *Max void size - upper statistical limit on defect size for a specified probability/confidence*
  - Estimation of risk of as-built foam divoting.





# ***TPS foam dissection database***



## ***What is being measured?***

- ***Foam Defects***
  - Defects due to foam chemistry (e.g. elongated cells)
  - Defects due to foam spray process (e.g. voids, rollovers)
- ***Focus of the analysis is characteristic defect size (CDS)***
  - Defects are measured in three dimensions:  $L1 > L2 > L3$
  - Slots are defined by  $L1 / L2 > 2.5$ ,  $CDS = L2$
  - Cylinders are defined by  $L1 / L2 < 2.5$ ,  $CDS = L1$

## ***Available dissection data***

PALRamps: 94LH<sub>2</sub>, 94LO<sub>2</sub>, 123LO<sub>2</sub>, 120LH<sub>2</sub>(10'), 121LH<sub>2</sub>(10'), 94SRB

I/T Flanges: 94LO<sub>2</sub>, 94LH<sub>2</sub>, 120LH<sub>2</sub>, 121LH<sub>2</sub>

Flange Thrust Panel 94LH<sub>2</sub>, 120LH<sub>2</sub>

LO2 feedline fairing: 94LO<sub>2</sub>, 120LH<sub>2</sub>, 121LH<sub>2</sub>

Feedline Yoke: 94 (incomplete)

Ice Frost Ramps: 94LO<sub>2</sub>, 128LO<sub>2</sub>, 94I/T,  
94LH<sub>2</sub>, 120LH<sub>2</sub>

Dog Leg C/T cover: 121LH<sub>2</sub>

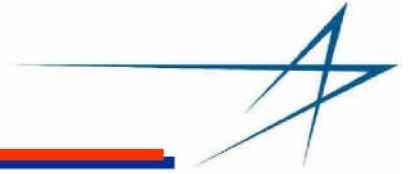
Longeron: 94LH<sub>2</sub>

Nosecone: 94

**Being updated**



# ***Assumptions and Limitations***



## ***Major Assumptions***

- The defects found in ET-94 are representative of all defects that are produced during the manually-sprayed processes for all in-stock tanks.
- The distributions of defects, for both slots and cylinders, are homogeneous (stable) and the samples are independent.

## ***Major Limitations to Generalization From ET-94***

- Data for some parts are available from only one tank (tank to tank variability needs to be expressed in the data)
- Data in some cases are a mixture of BX-250 and BX-265 material
- Data on some parts have very small sample size (e.g., N=3).
- Samples are not random —Tank 94 plus certain parts of other tanks removed in support of redesign
- Sprayer to sprayer differences are not well understood.



## **Assumptions and Limitations (cont.)**



### ***Major Limitations of Fitting Distribution Shapes To Defect Size***

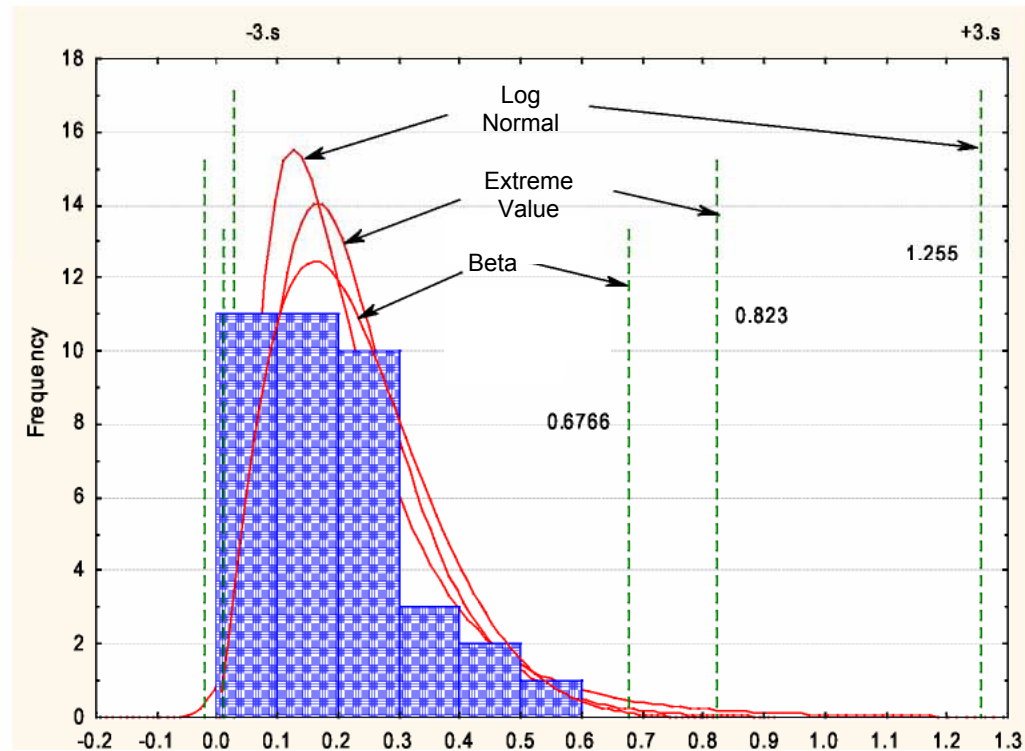
- It is not unusual to fit distribution shapes to data, under conditions of uncertainty. However, the level of uncertainty related to distribution shape is high for defect CDS.
- Distribution related limitations for manually-sprayed foam dissections
  - *There is no a priori knowledge of the shape of distribution of defect CDS.*
  - *There is no engineering rationale to pick a specific distribution.*
  - *The natural variation of the process is not well understood.*
  - *There is a lack of random samples (historical) of sufficient size to empirically select a distribution*
  - *Process controls related to manually-sprayed foam were related to environmental parameters and overlap time. Distributions of these process variables have been studied. However, the relationship between process control variables and defects is not known.*
  - *Process controls for manually-sprayed foam were related to environmental parameters and overlap time, but other technique related variables were not controlled.*



# Example of Fitting a Standard Distribution



- For predicting the maximum CDS, the critical part of the distribution is the tail, where we have the least information.
- Standard goodness-of-fit tests are not sensitive to shape of the tail of the distribution.
- For the CDS datasets provided, most datasets can be fit well by several distributions having very different tail properties — different “3-sigma” values.




- It is common practice to pick the best fit. However, picking a distribution, determines the “3-sigma” value.
- A distribution that does not statistically fit a set of data could be the distribution that best represents the shape of the tail



## **Conclusion and Recommendations**



- 1. Dissection data statistics have severe limitations and significant uncertainty in predicting the size of the largest defect CDS on the next tank.***
- 2. Any statistical results should be subject to an engineering evaluation for consistency with experience, engineering analysis, and hardware limitations***
- 3. Statistical analysis results/estimates should be used in a support role and as information to engineering in ET certification***
- 4. Data limitations need to be addressed and minimized to improve the credibility of the statistical analysis to support RTF decisions (data must address tank to tank or part to part variation)***
- 5. The ET Program needs to pursue methods beside statistics to predict the largest possible defect CDS.***

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## Appendix C. Statistical Methodology

Consider these two random variables:

- 1) The measurement of a single foam defect, selected at random.
- 2) The measurement of N foam defects, and the selection of the maximum of the N values.

If  $N > 1$ , these variables are not the same, in particular, the equivalent 3-sigma point in the distribution of (1) can be much less than that of (2). If one is trying to ensure that none of the foam defects exceed a critical size, probability statements about variable (2) are needed to characterize the risk.

### Predicting the maximum Characteristic Defect Size for the next tank.

The largest CDS value in the next sample is the quantity of interest. The distribution of the maximum value of a set of random variables can be derived from their individual distributions. The result is most easily described in terms of Cumulative Distribution Functions (CDF). If the Probability Distribution Function (PDF) of X is  $P(x)$ , the CDF is defined to be the probability that the measured value is  $\leq$  to x:

$$CDF(x) = \int_0^x P(x') dx'$$

Given a set of N independent variables, say  $\mathbf{X} = \{ X_1, X_2, \dots, X_N \}$ , the CDF of their maximum is the product of their CDFs:

$$CDF(\max(\mathbf{X})) = CDF(X_1) CDF(X_2) \dots CDF(X_N)$$

so if the X's are N random “draws” from the same distribution, this equation becomes

$$CDF(\max(\mathbf{X})) = CDF(X)^N$$

The figure on the following page shows PDFs (derivatives of the CDFs) for the maximum of N variables, normally distributed – with zero mean and unit variance. The normal distribution fit to the logs of the defect sizes is one of the candidate distributions. Distributions of maximums can be derived for any candidate distribution.





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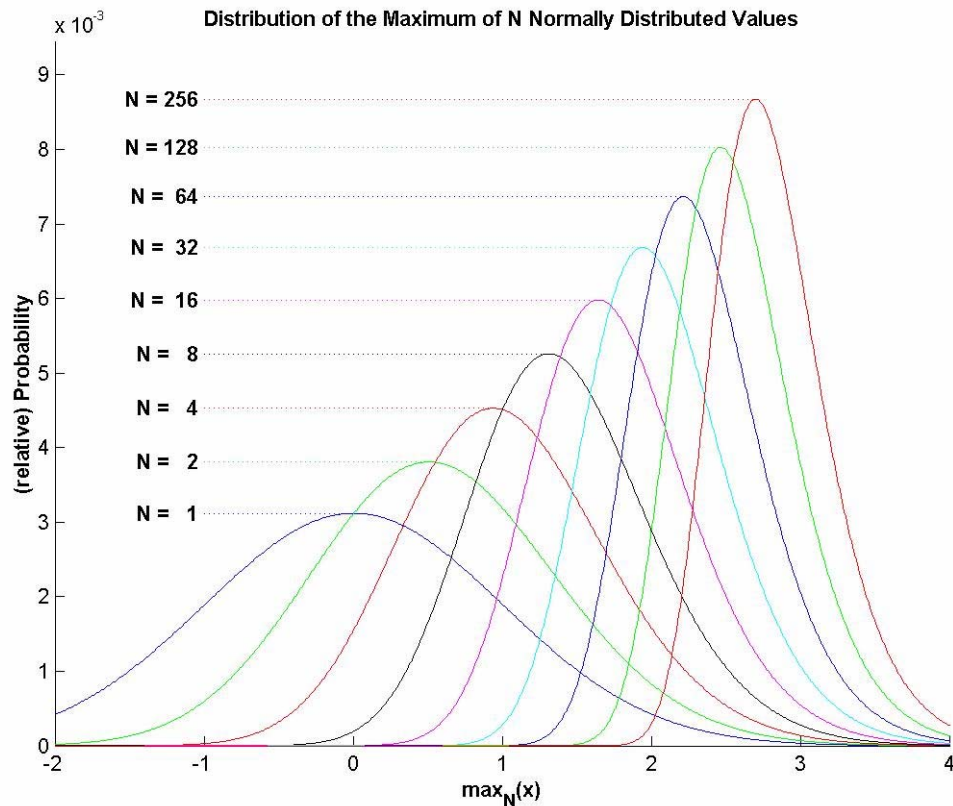
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**1.0**

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The curve for  $N=1$  is just the normal distribution itself. For increasing  $N$ s, the peak of the curve (the most likely value of the maximum of  $N$  draws) shifts to larger values as one would expect, because, with more tries, the relatively rare values on the high-end of the distribution are more likely to be obtained. So, the distribution of the maximum values of a set of samples increases with the numbers of measurements in a sample. The variance of distribution of maximums decreases with  $N$ .

The equation of interest is:

$$P(\text{next}_{\max} > \text{critical value}) = P_{\text{crit}} = 1 - \text{CDF}(X)^N$$


Either we set a  $P_{\text{crit}}$ , for example to a probability equivalent 3-sigma for the normal (1-.9987) and solve for a critical value; or we can determine the probability that the  $\text{next}_{\max}$  is greater than some critical value—in this case a value set by engineering.

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The population equivalent 3-Sigma point of a single defect CDS provides an estimate such that 99.87% of the time a randomly chosen defect from the next tank will be less than this value. This estimate does not change with the expected number of defects in a specific part. Using the relationship between a single sample distribution and the distribution of maximums, the probability that the maximum of the next sample is greater than the single defect CDS equivalent 3-Sigma point conditional on sample size can be determined. That is, the .13% chance of seeing a larger maximum defect could actually be a 12% chance if 100 defects were expected.

Expected N	P(observed max > 3 sigma single point estimate)
1	0.0013
2	0.002598
5	0.006483
10	0.012924
20	0.025681
30	0.038274
40	0.050703
50	0.062972
60	0.075083
70	0.087036
80	0.098836
90	0.110483
100	0.121979
200	.33907883
400	.40568054
600	.54182654



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Title: <b>Support Provided to the External Tank (ET) Project on the Use of Statistical Analysis for ET Certification</b>			Page #: 37 of 37

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14. ABSTRACT In June 2004, the June Space Flight Leadership Council (SFLC) assigned an action to the NASA Engineering and Safety Center (NESC) and External Tank (ET) project jointly to characterize the available dataset [of defect sizes from dissections of foam], identify resultant limitations to statistical treatment of ET as-built foam as part of the overall thermal protection system (TPS) certification, and report to the Program Requirements Change Board (PRCB) and SFLC in September 2004. The NESC statistics team was formed to assist the ET statistics group in August 2004. The NESC's conclusions are presented in this report.						
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